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Generalized supersymetric boundary state

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Abstract: Following our previous paper (hep-th/9909027), we generalize a supersymmetric boundary state so that arbitrary configuration of the gauge field coupled to the boundary of the worldsheet is incorpolated. This generalized boundary state is BRST invariant and satisfies the non-linear boundary conditions with non-constant gauge field strength. This boundary state contains divergence which is identical with the loop divergence in a superstring σ model. Hence vanishing of the β function in the superstring σ model corresponds to a well-defined boundary state with no divergence. The coupling of a single closed superstring massless mode with multiple open string massless modes is encoded in the boundary state, and we confirm that derivative correction to the D-brane action in this sector vanishes up to the first non-trivial order $O(\alpha'\partial^2)$. Combining T-dualities, we incorpolate also general configurations of the scalar fields on the D-brane, and construct boundary states representing branes stuck to another D-brane, with use of BIon configuration.

Keywords: Superstrings and Heterotic Strings, D-branes.

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Contents

1. Introduction

Recent development on D-branes in string theories is owing in some part to the analysis of the boundary state [1, 2, 3, 4, 5, 6]. This ingredient is effective to describe boundary of string worldsheet, especially when it is studied in the ordinary parturbative approach of closed string conformal field theory. The boundary state itself has been developed to formulate string loop corrections to the effective equations of motion of string theory in relation to the Fischler-Susskind mechanism [1, 2, 3]. In most of the studies of the boundary state, only the constant gauge field strength was taken into account as the boundary degree of freedom. This is because in that case the boundary conditions satisfied by the coordinate fields on the worldsheet become linear, hence one can construct explicitly the eigen state of the equations of the boundary conditions.

The D-brane [7] opened new directions on the use of the boundary state. Since this state specifies the couplings between D-brane dynamical degrees of freedom and closed string excitations, the state is useful for deriving D-brane actions, the effective actions of the D-branes [8, 9]. The D-brane actions have been indicative of various dualities in string theories [10]. However, as mentioned above, these D-brane actions have been obtained in only the leading order estimation with respect to the derivatives acting on the fields on the D-branes. The derivative corrections may ruin the intriguing properties which the D-brane actions possess, and hence a systematic method for evaluating higher order corrections of the D-brane actions and the boundary states are needed [11, 12, 13].

In this paper, we extend our previous result of ref. [14] to the superstring case. We construct in superstring theory a generalized boundary state which incorporates whole degrees of freedom of the boundary-coupled background fields, which are the gauge fields and the scalar fields on the D-brane. This boundary state is BRST invariant and reduces to the previously known form when the field strength is put constant. Using the boundary state, we can calculate particular terms in the D-brane action: the terms linear in closed string massless modes. By generalizing the boundary state, we obtain corrections to the relevant part of the D-brane action. This method may give a gleam of hope to understand the string dualities to all order.

The organization of this paper is as follows. In section 2, we obtain a supersymmetric extension of the generalized boundary state given in our previous paper [14] in which in the bosonic string theory a boundary gauge field (in particular, non-constant modes of the field strength) was incorporated to the boundary state. We treat type-II superstring theory in this paper. After checking the BRST invariance of the generalized boundary state, in section 3 we evaluate the divergence immanent in this state. The definition of the generalized boundary state includes products of the coordinate scalar field on a single worldsheet point, thus contains the normal-ordering divergence. In the evaluation of the divergence, we adopt the approximation of slowly-varying fields. This divergence is found to be identical to the superstring σ model loop divergence, hence the vanishing of the β function in the string σ model corresponds to a well-defined boundary state with no divergence. Then with use of this well-defined state we find that there is no finite correction to the D-brane action up-to this order $O(\alpha'\partial^2)$ in the sector of the coupling linear in massless closed string mode. This result is in contrast to the bosonic string case [14], in which the finite corrections exist. In section 4, utilizing T-duality transformation we introduce scalar fields into the boundary state, and see that there is no derivative correction to the D-brane action even with the scalar fields. In appendix A, we present detailed calculations which are necessary in section 3. In appendix B, we apply our generalized boundary state to the BIon configuration. We see that the "spike" part of the BIon actually describes the stirng stuck to the D-brane.

2. Generalized supersymmetric boundary state

As mentioned in the introduction, most aspects of the D-branes are concerning supersymmetry and their BPS properties. Hence the low energy effective actions of the D-branes are desirably studied in a supersymmetric fashion. In this section, we apply the idea developed in our previous paper [14] to superstring theory and construct a generalized supersymmetric boundary state, incorporating the non-constant gauge field strength on the D-brane. The scalar field which also exists on the D-brane shall be treated later in section 4.

2.1 Review of the generalized bosonic boundary state

First let us summarize the definition and the relevant properties of the generalized boundary state with the non-constant field strength in bosonic string theory [14]. In the case of constant gauge field strength $F_{\mu\nu}$, the boundary state $|B^{(b)}(F)\rangle$ is defined as an eigen state of the linear boundary conditions for open bosonic strings [1, 2, 4]:

$$X^i(\sigma)\left|B^{(b)}(F)\right\rangle = 0\,, \tag{2.1}$$

$$(\pi P_\mu(\sigma) + F_{\mu\nu}\partial_\sigma X^\nu(\sigma))\left|B^{(b)}(F)\right\rangle = 0\,, \tag{2.2}$$

$$\pi_c(\sigma)\left|B^{(b)}(F)\right\rangle = \pi_{\bar{c}}(\sigma)\left|B^{(b)}(F)\right\rangle = 0\,. \tag{2.3}$$

The index i specifies the directions transverse to the D-brane, while the Greek indices μ,ν run in the longitudinal directions. The superscript (b) denotes that this is for the bosonic part. The BRST invariance of this boundary state is a consequence of eqs. (2.1)–(2.3). The oscillator representation of $\left|B^{(b)}(F)\right\rangle$ reads

$$\left|B^{(b)}(F)\right\rangle = -\frac{T_p}{4}N(F)\,|B_{\rm N}(F)\rangle\otimes|B_{\rm D}\rangle\otimes|B_{\rm gh}\rangle\,, \tag{2.4}$$

where three factors of kets $|B_{\rm N}(F)\rangle$, $|B_{\rm D}\rangle$ and $|B_{\rm gh}\rangle$ are for satisfying (2.1), (2.2) and (2.3), respectively. These are given by

$$|B_{\rm N}(F)\rangle = \exp\left\{-\sum_{n\geq 1}\frac{1}{n}\alpha^{(-)\mu}_{-n}\mathcal{O}(F)_\mu{}^\nu\alpha^{(+)}_{-n\nu}\right\}|0\rangle_{p+1}\,, \tag{2.5}$$

$$|B_{\rm D}\rangle = \exp\left\{\sum_{n\geq 1}\frac{1}{n}\alpha^{(-)i}_{-n}\alpha^{(+)}_{-ni}\right\}|0\rangle_{d-p-1}\,\delta^{d-p-1}(x^i)\,, \tag{2.6}$$

$$|B_{\rm gh}\rangle = \exp\left\{\sum_{n\geq 1}\left(c^{(-)}_{-n}\,\bar{c}^{(+)}_{-n} + c^{(+)}_{-n}\,\bar{c}^{(-)}_{-n}\right)\right\}|0\rangle_{\rm gh}\,. \tag{2.7}$$

In eq. (2.5), The orthogonal matrix $\mathcal{O}$ is defined as $\mathcal{O}_\mu{}^\nu = (1-F)_\mu^\rho\{(1+F)^{-1}\}_\rho{}^\nu$. The front factor $N(F)$ is obtained in various ways [2, 3, 15, 16] as

$$N(F) = \Big(\det(1+F)\Big)^{-\zeta(0)}\,. \tag{2.8}$$

When the front factor is put in this form (2.8), this boundary state can be rewritten in a form in which the dependence on the gauge field is simply arranged:

$$\left|B^{(b)}(F)\right\rangle = \exp\left(\frac{i}{2\pi}\oint d\sigma\, \partial_\sigma X^\mu F_{\mu\nu} X^\nu\right)\left|B^{(b)}(F=0)\right\rangle . \tag{2.9}$$

In ref. [14], we have generalized this boundary state to the one with general configuration of the gauge field, using the properties of the star product (three-string interaction vertex) in a string field theory. The definition of this generalized boundary state is

$$\left|\mathcal{B}^{(b)}\left(A_\mu(x)\right)\right\rangle \equiv U[A]\left|B^{(b)}(F=0)\right\rangle , \tag{2.10}$$

with

$$U[A] \equiv \exp\left(\frac{-i}{\pi}\oint d\sigma \partial_\sigma X^\mu A_\mu(X)\right). \tag{2.11}$$

One observes that, for the constant field strength, the generalized boundary state (2.10) is reduced to the previous one (2.9). This boundary (2.10) state is subject to the following non-linear boundary condition in the longitudinal directions

$$\left[\pi P_\mu + F_{\mu\nu}(X)\partial_\sigma X^\nu\right]\left|\mathcal{B}^{(b)}\right\rangle = 0 , \tag{2.12}$$

in addition to the usual Dirichlet boundary conditions $X^i\left|\mathcal{B}^{(b)}\right\rangle = 0$. Note that in eq. (2.12), the field strength can be general, not only constant. Furthermore, we can show that this $|\mathcal{B}\rangle$ is BRST invariant,

$$Q_{\mathrm{B}}\left|\mathcal{B}^{(b)}\right\rangle = 0 , \tag{2.13}$$

with use of the boundary conditions above.

2.2 Supersymmetric extension

In the Neveu-Schwarz-Ramond (NS-R) formulation of superstring theory, worldsheet fermions and superghosts are introduced in addition to the contents in bosonic string theory. In this paper we shall not mention the superghost part in the boundary state explicitly, since this is not concerned with the gauge field configuration.

In the absence of non-trivial gauge field on the boundary, the fermionic boundary conditions for the superstring worldsheet are $(\psi_+ \mp i\psi_-)^\mu = 0$ in the directions tangential to the D-brane, and $(\psi_+ \pm i\psi_-)^i = 0$ for the transverse directions [1, 2, 4]. The sign $\pm$ is the spin structure, and we leave it undetermined.[1] Therefore the fermionic sector of the boundary state $|\mathcal{B}^{(f)}(A_\mu(x))\rangle$ should satisfy

$$(\psi_+ \mp i\psi_-)^\mu\left|\mathcal{B}^{(f)}(A=0)\right\rangle = (\psi_+ \pm i\psi_-)^i\left|\mathcal{B}^{(f)}(A=0)\right\rangle = 0 . \tag{2.14}$$

[1] Our result on the correction to the D-brane action does not refer to the spin structure as well as other fine structures such as GSO projections.

The explicit oscillator representation of this eigen state is

$$
\left|\mathcal{B}^{(f)}(A=0)\right\rangle = \exp\left\{ \mp i \sum_{r>0} b_{-r}^{(-)\mu}\, b_{-r\mu}^{(+)} \right\} \times \\
\times \exp\left\{ \pm i \sum_{r>0} b_{-r}^{(-)i}\, b_{-ri}^{(+)} \right\} \left|B_0^{(f)}\right\rangle \otimes \left|B_{\text{superghost}}\right\rangle , \qquad (2.15)
$$

where $|B_0^{(f)}\rangle$ is the fermionic zero-mode part which exists only in the R-R sector. It is given as

$$
\left|B_0^{(f)}\right\rangle \equiv \Pi_i \theta^i \left|0;\pm\right\rangle , \qquad (2.16)
$$

where we have defined a linear combination of the fermionic zero modes $\theta^M \equiv \psi_{0+}^M \pm i\psi_{0-}^M$. The superghost sector $|B_{\text{superghost}}\rangle$ is given in ref. [1].

Now, let us define the fermionic part of the generalized boundary state as

$$
\left|\mathcal{B}^{(f)}\Big(A_\mu(x)\Big)\right\rangle \equiv U[F]\left|\mathcal{B}^{(f)}(A=0)\right\rangle , \qquad (2.17)
$$

where the unitary operator $U[F] \equiv \exp R$ is given by

$$
R \equiv \frac{1}{8\pi}\oint d\sigma(\psi_+ \pm i\psi_-)^\mu(\psi_+ \pm i\psi_-)^\nu F_{\mu\nu}[X] . \qquad (2.18)
$$

From the definition, this boundary state incorporates general configuration of the gauge field strength $F_{\mu\nu}[X]$ and evidently gauge invariant. Since the operator $U[F]$ is subject to the following relation

$$
U[F](\psi_+ \mp i\psi_-)^\mu U[F]^{-1} = (\psi_+ \mp i\psi_-)^\mu - F^\mu{}_\nu[X](\psi_+ \pm i\psi_-)^\nu , \qquad (2.19)
$$

then for the general background we obtain the non-linear boundary condition for the tangential directions as

$$
\Big[(\psi_+ \mp i\psi_-)^\mu - F^\mu_\nu[X](\psi_+ \pm i\psi_-)^\nu\Big]\left|\mathcal{B}^{(f)}\Big(A_\mu(x)\Big)\right\rangle = 0 . \qquad (2.20)
$$

This boundary condition is actually the one given in ref. [17], from the supersymmetrization of the boundary coupling in the string σ model.

Combining this fermionic part with the bosonic part in the previous subsection, the total unitary operator becomes $U \equiv U[A]U[F]$. Since we have introduced general X dependence in the fermionic unitary operator $U[F]$, the boundary condition for the bosonic part (2.12) is changed. Actually, using the relation

$$
[\pi P_\rho, R] = \frac{-i}{8}(\psi_+ \pm i\psi_-)^\mu(\psi_+ \pm i\psi_-)^\nu \frac{\delta}{\delta X^\rho} F_{\mu\nu}[X] , \qquad (2.21)
$$

we obtain

$$
U[A]U[F](\pi P_\mu)U[F]^{-1}U[A]^{-1} = \\
= \pi P_\mu + F_{\mu\nu}[X]\partial_\sigma X^\nu + \frac{i}{8}\partial_\mu F_{\nu\rho}[X](\psi_+ \pm i\psi_-)^\nu(\psi_+ \pm i\psi_-)^\rho , \qquad (2.22)
$$

thus the boundary condition is modified by the third term on the right-hand side. We summarize the boundary conditions as

$$\left[\pi P_\mu + F_{\mu\nu}[X]\partial_\sigma X^\nu + \frac{i}{8}\partial_\mu F_{\nu\rho}[X](\psi_+ \pm i\psi_-)^\nu(\psi_+ \pm i\psi_-)^\rho\right]|\mathcal{B}\rangle = 0\,, \quad (2.23)$$

$$\left[(\psi_+ \mp i\psi_-)^\mu - F^\mu{}_\nu[X](\psi_+ \pm i\psi_-)^\nu\right]|\mathcal{B}\rangle = 0\,, \quad (2.24)$$

where we have defined the complete boundary state as

$$|\mathcal{B}\rangle \equiv \left|\mathcal{B}^{(b)}\right\rangle \otimes \left|\mathcal{B}^{(f)}\right\rangle\,. \quad (2.25)$$

Of course when the field strength is constant, the generalized boundary state (2.25) reproduces the usually utilized boundary state. For the bosonic sector of the boundary state, this is checked in ref. [14]. For the fermionic part, exponentiating the linear operator R (2.18), we have an explicit form of the boundary state which is written only by the creation operators as

$$\left|B^{(f)}(F)\right\rangle = N^{(f)}(F)\exp\left\{\mp i\sum_{r>0} b_{-r}^{(-)\mu}\mathcal{O}_\mu^\nu\, b_{-r\nu}^{(+)}\right\}\left|B_0^{(f)}(F)\right\rangle \otimes |B_{\text{superghost}}\rangle\,. \quad (2.26)$$

In the R-R sector, the fermionic zero mode is modified and depends on the constant field strength as

$$\left|B_0^{(f)}(F)\right\rangle \equiv \exp\left(\frac{1}{4}\theta^\mu\theta^\nu F_{\mu\nu}\right)\Pi_i\theta^i\,|0;\pm\rangle\,. \quad (2.27)$$

The normalization factor $N^{(f)}(F)$ is given by

$$N^{(f)}(F) = \begin{cases} 1 & \text{(for the NS-NS sector)}\,, \\ (\det(1+F))^{\zeta(0)} & \text{(for the R-R sector)}\,. \end{cases} \quad (2.28)$$

This resultant form coincides with the one given in ref. [2].

2.3 BRST invariance of the boundary state

The annihilation of the BRST operator on the boundary state is important for the string loop correction to the equation of motion in superstring theory [1]. In this subsection we show that the generalized boundary state constructed in the previous section is indeed BRST invariant.

The BRST charge is composed of relevant two terms in addition to the pure ghost sector as

$$Q_{\mathrm{B}} = Q_{\mathrm{B}}^{(L)} + Q_{\mathrm{B}}^{(F)} + (\text{ghost sector})\,, \quad (2.29)$$

where

$$Q_{\mathrm{B}}^{(L)} \equiv \sum_\pm\sum_n L_{-n}^{\alpha,b}c_n\,, \quad Q_{\mathrm{B}}^{(F)} \equiv \sum_\pm\sum_n F_{-n}^{\alpha,b}\gamma_n\,, \quad (2.30)$$

and we see the vanishing of these two operators $Q_{\rm B}^{(L)}$ and $Q_{\rm B}^{(F)}$, respectively. For the former term $Q_{\rm B}^{(L)}$, the following expression is found to be useful:

$$Q_{\rm B}^{(L)} = \frac{1}{\sqrt{\pi}} \oint \left[i\Pi_{\bar{c}} \{\cdots\} + c\Big\{ 2\pi P_M \partial_\sigma X^M + \frac{i}{2} \left(\psi_+ \partial_\sigma \psi_+ + \psi_- \partial_\sigma \psi_- \right) \Big\} \right]. \tag{2.31}$$

The first term in the integrand disappears due to the ghost boundary condition $\Pi_{\bar{c}} = 0$. Next, the term $P_M \partial_\sigma X^M$ is evaluated using the boundary condition (2.23) as

$$\begin{aligned} 2\pi P_M \partial_\sigma X^M \, |\mathcal{B}\rangle &= 2\Big[- F_{\mu\nu}[X] \partial_\sigma X^\nu - \frac{i}{8} \partial_\mu F_{\nu\rho}[X] \times \\ &\qquad \times (\psi_+ \pm i\psi_-)^\nu (\psi_+ \pm i\psi_-)^\rho \Big] \partial_\sigma X^\mu \, |\mathcal{B}\rangle = \\ &= -\frac{i}{4} \partial_\mu F_{\nu\rho}[X] (\psi_+ \pm i\psi_-)^\nu (\psi_+ \pm i\psi_-)^\rho \partial_\sigma X^\mu \, |\mathcal{B}\rangle \, . \end{aligned} \tag{2.32}$$

For the evaluation of the rest term in eq. (2.31), we use a new condition satisfied by $|\mathcal{B}\rangle$,

$$\Big[\partial_\sigma (\psi_+ \mp i\psi_-)^\mu - F^\mu_\nu[X] \partial_\sigma (\psi_+ \pm i\psi_-)^\nu - \partial_\sigma X^\rho \partial_\rho F^\mu{}_\nu[X] (\psi_+ \pm i\psi_-)^\nu \Big] \, |\mathcal{B}\rangle = 0 \, , \tag{2.33}$$

which is derived by differentiating the fermionic boundary condition (2.24) by σ. Then we obtain

$$\begin{aligned} &\frac{i}{2} \Big[\psi_+ \partial_\sigma \psi_+ + \psi_- \partial_\sigma \psi_- \Big] \, |\mathcal{B}\rangle = \\ &\quad = \frac{i}{2} \left[\frac{1}{2} (\psi_+ \mp i\psi_-)^\mu \partial_\sigma (\psi_+ \pm i\psi_-)_\mu + \frac{1}{2} (\psi_+ \pm i\psi_-)^\mu \partial_\sigma (\psi_+ \mp i\psi_-)_\mu \right] |\mathcal{B}\rangle = \\ &\quad = \frac{i}{4} \partial_\mu F_{\nu\rho}[X] (\psi_+ \pm i\psi_-)^\nu (\psi_+ \pm i\psi_-)^\rho \partial_\sigma X^\mu \, |\mathcal{B}\rangle \, , \end{aligned} \tag{2.34}$$

which exactly cancels the term from $P_M \partial_\sigma X^M$, (2.32). Hence the operator $Q_{\rm B}^{(L)}$ vanishes on the generalized boundary state (2.25).

Another operator $Q_{\rm B}^{(F)}$ is expressed as

$$Q_{\rm B}^{(F)} = \frac{1}{2\pi} \oint d\sigma \Big[(\pi P_M - \partial_\sigma X_M) \psi_+^M \gamma_+ + (\pi P_M + \partial_\sigma X_M) \psi_-^M \gamma_- \Big]. \tag{2.35}$$

Using the boundary conditions for the superghost [1]

$$\gamma_+ = \mp i \gamma_- \, , \tag{2.36}$$

in addition to the boundary conditions (2.23) and (2.24), we obtain

$$Q_{\rm B}^{(F)} \, |\mathcal{B}\rangle = \frac{1}{2\pi} \oint \left[-\frac{i}{8} \partial_\mu F_{\nu\rho} (\psi_+ \pm i\psi_-)^\mu (\psi_+ \pm i\psi_-)^\nu (\psi_+ \pm i\psi_-)^\rho \gamma_+ \right] |\mathcal{B}\rangle \, , \tag{2.37}$$

which vanishes due to the Bianchi identity.

Summing up all together, we see the BRST invariance of the generalized boundary state $|\mathcal{B}\rangle$. Since in the generalized boundary state full configuration of the gauge field coupled to the boundary is included, this degree of freedom of the gauge field is found to be actually one of the collective coordinates of the BRST equation. However, any configuration of the gauge field is not allowed, since this boundary state contains divergence in general. For special configuration of the gauge field, the boundary state is not divergent and well defined, as in the case of the bosonic string theory in ref. [14]. We will see this in the next section.

3. Correction to the D-brane action

In this section, we calculate the corrections to the D-brane action, using the supersymmetric generalized boundary state. The procedure was already explored in our previous paper [14], and here we shall extend the calculation given there to the supersymmetric case. We expand the field strength by derivatives, whose number is at most two. Therefore we are considering the first correction to the D-brane action, that is $O(\alpha'\partial^2)$, in the slowly-varying field approximation.

3.1 Divergence in the boundary state

Before proceeding to the explicit evaluation of the correction to the D-brane action, we explain how to calculate the relevant parts of the generalized boundary state.

As seen in the definition of the operator in eqs. (2.11) and (2.18), the newly defined boundary state (2.25) contains short distance divergence on the worldsheet. Therefore, before calculating the correction to the D-brane action, we must obtain well-defined boundary state with no divergence, through a certain "renormalization" procedure.

It would be hard to evaluate the operator $U[A]$ and $U[F]$ without using the explicit functional form of the gauge field. Here we adopt the Taylor expansion for the gauge field, and ignore the higher derivative terms. We keep only the terms in which the number of the derivatives acting on the gauge field strength is at most two. Then in this approximation, we can extract the short distance divergence (or normal-ordering divergence) which exists in the boundary state, with use of the simple linear boundary conditions. Interestingly, the divergence in the generalized boundary state can be perfectly absorbed into the redefinition of the gauge field:

$$A_\mu^{\rm red}(x) \equiv A_\mu(x) + \beta[A]\zeta(1)\,. \tag{3.1}$$

The boundary state can be described by creation operators and this redefined gauge field with finite coefficients. Of course, this redefinition includes the normal ordering divergence $\zeta(1)$. For obtaining the well-defined boundary state, we are forced to put the coefficient of the divergent part to be zero: $\beta[A] = 0$. This is the "renormalization" procedure for obtaining the well-defined boundary state.

After putting this constraint on the gauge field, we can proceed to the evaluation of the correction to the D-brane action. In the following, after the explicit calculation, we will see that the functional $\beta[A]$ in the above redefinition (3.1) is actually identical to the familiar beta function in the superstring σ model. Thus we interpret this constraint as the worldsheet conformal invariance constraint.

These facts which will be explained in detail below are just analogous to the situation of the generalized boundary state in the bosonic string theory explored in ref. [14].

Now, let us present the detailed calculation. First, write explicitly the zero mode part of the coordinate scalar and fermions as

$$X^\mu(\sigma) = x^\mu + \widetilde{X}^\mu(\sigma)\,, \qquad \psi^\mu_\pm = \psi^\mu_{0\pm} + \widetilde{\psi}_\pm\,. \tag{3.2}$$

The zero mode of the fermion $\psi^\mu_{0\pm}$ exists only in the Ramond sector. The gauge field depends arbitrarily on X and we Taylor-expand $A_\mu\left(X(\sigma)\right)$ around the zero mode x^μ:

$$A_\mu(X) = A_\mu(x) + \widetilde{X}^\nu \partial_\nu A_\mu(x) + \frac{1}{2}\widetilde{X}^\nu \widetilde{X}^\rho \partial_\nu \partial_\rho A_\mu(x) + \cdots\,, \tag{3.3}$$

$$F_{\mu\nu}(X) = F_{\mu\nu}(x) + \widetilde{X}^\rho \partial_\rho F_{\mu\nu}(x) + \frac{1}{2}\widetilde{X}^\rho \widetilde{X}^\delta \partial_\rho \partial_\delta F_{\mu\nu}(z) + \cdots\,. \tag{3.4}$$

We decompose the unitary operator concerning the zero modes of X as

$$U[A] = V U_0\,, \qquad U[F] = V^{(\mathrm{f})} U_0^{(\mathrm{f})}\,, \tag{3.5}$$

where

$$U_0 = \exp\left(\frac{i}{2\pi} F_{\mu\nu}(x) \oint d\sigma \partial_\sigma \widetilde{X}^\mu \widetilde{X}^\nu\right), \tag{3.6}$$

$$U_0^{(\mathrm{f})} = \exp\left(\frac{1}{8\pi} F_{\mu\nu}(x) \oint d\sigma (\psi_+ \pm i\psi_-)^\mu (\psi_+ \pm i\psi_-)^\nu\right). \tag{3.7}$$

The rest part is written in an expanded form as

$$\begin{aligned} V = {} & 1 + \frac{-i}{3\pi}\partial_\rho F_{\nu\mu}(x) \oint d\sigma \partial_\sigma \widetilde{X}^\mu \widetilde{X}^\nu \widetilde{X}^\rho + \frac{-i}{8\pi}\partial_\delta \partial_\rho F_{\nu\mu}(x) \oint d\sigma \partial_\sigma \widetilde{X}^\mu \widetilde{X}^\nu \widetilde{X}^\rho \widetilde{X}^\delta + \\ & + \frac{1}{2}\left(\frac{-i}{3\pi}\partial_\rho F_{\nu\mu}(x) \oint d\sigma \partial_\sigma \widetilde{X}^\mu \widetilde{X}^\nu \widetilde{X}^\rho\right)^2 + \cdots\,, \end{aligned} \tag{3.8}$$

$$\begin{aligned} V^{(\mathrm{f})} = {} & 1 + \frac{1}{8\pi}\partial_\rho F_{\mu\nu}(x) \oint d\sigma (\psi_+ \pm i\psi_-)^\mu (\psi_+ \pm i\psi_-)^\nu \widetilde{X}^\rho + \\ & + \frac{1}{8\pi}\partial_\delta \partial_\rho F_{\mu\nu}(x) \oint d\sigma (\psi_+ \pm i\psi_-)^\mu (\psi_+ \pm i\psi_-)^\nu \widetilde{X}^\rho \widetilde{X}^\delta + \\ & + \frac{1}{2}\left(\frac{1}{8\pi}\partial_\rho F_{\mu\nu}(x) \oint d\sigma (\psi_+ \pm i\psi_-)^\mu (\psi_+ \pm i\psi_-)^\nu \widetilde{X}^\rho\right)^2 + \cdots\,. \end{aligned} \tag{3.9}$$

In the above eqs. (3.8) and (3.9) we have kept terms in which the total number of the derivatives acting on $F_{\mu\nu}(x)$ is at most two, as mentioned before. This is for the leading order derivative correction to the D-brane action, of $O(\alpha'\partial^2)$.

Note that $[U_0, V^{(\mathrm{f})}] = 0$ and that the zero mode contribution in the unitary operator is summarized as

$$U_0 U_0^{(\mathrm{f})} \left|B(F=0)\right\rangle = \left|B\Big(F(x)\Big)\right\rangle . \tag{3.10}$$

Here we have defined the boundary state for the constant field strength by eqs. (2.4) and (2.26) as

$$\left|B(F)\right\rangle \equiv \left|B^{(b)}(F)\right\rangle \otimes \left|B^{(f)}(F)\right\rangle , \tag{3.11}$$

and the quantity $|B(F(x))\rangle$ appearing in eq. (3.10) is obtained by substituting $F(x)$ into the place of the constant F in $|B(F)\rangle$. From these relations we obtain an expression

$$|\mathcal{B}\rangle = V V^{(\mathrm{f})} \left|B\Big(F(x)\Big)\right\rangle . \tag{3.12}$$

The base state $|B(F(x))\rangle$ is written explicitly only by the creation operators (see eq. (2.26)), and satisfies the following boundary conditions

$$\left(\alpha_{n\mu}^{(-)} + \mathcal{O}_\mu^\nu(x)\alpha_{-n\nu}^{(+)}\right)\left|B\Big(F(x)\Big)\right\rangle = 0 , \tag{3.13}$$

$$\left(b_{r\mu}^{(-)} \pm i\mathcal{O}_\mu^\nu(x) b_{-r\nu}^{(+)}\right)\left|B\Big(F(x)\Big)\right\rangle = 0 , \tag{3.14}$$

which relate the annihilation operators on $|B(F(x))\rangle$ to the creation operators. Here $\mathcal{O}(x)$ is an orthogonal matrix defined by

$$\mathcal{O}_\mu{}^\nu(x) \equiv \left(\frac{1-F(x)}{1+F(x)}\right)_\mu{}^\nu . \tag{3.15}$$

In order to extract the normal ordering divergence in the boundary state (3.12), we change all the annihilation operators in the derivative perturbation V into the creation ones with use of these boundary conditions (3.13) and (3.14).

We have accomplished this procedure for V and $V^{(f)}$ to the order mentioned above, and present here only the result (see appendix A for the detailed calculation). The result is the same as the case of the bosonic string theory [14]. The boundary state to this order, expressed by only the creation operators, contains $\zeta(0)$ and $\zeta(1)$ divergences. After regularizing one of the divergences as $\zeta(0) = -1/2$, we are left with an intrinsic short-distance divergence $\zeta(1)$. However, the $\zeta(1)$ divergent terms are arranged in a form in which the divergence is perfectly absorbed into the redefinition of the field strength $F(x)$ as

$$|\mathcal{B}\rangle = \left|B(F^{\mathrm{red}}(x))\right\rangle + \begin{bmatrix}\text{creation operators}\\ \text{with finite coefficients}\end{bmatrix} \left|B\Big(F(x)\Big)\right\rangle , \tag{3.16}$$

where the redefined field strength $F^{\rm red}(x)$ is given by

$$F^{\rm red}_{\mu\nu}(x) \equiv F_{\mu\nu}(x) + \frac{1}{2}\zeta(1)\left(\frac{1}{1+F}\right)^{\rho\delta}\partial_\rho\partial_\delta F_{\mu\nu} + \frac{1}{4}\zeta(1)\times$$
$$\times\left[\left\{\left(\frac{1}{1+F}\right)^{\lambda\rho}\partial_\nu F_{\rho\delta}\left(\frac{1}{1+F}\right)^{\delta\kappa}(\partial_\lambda F_{\kappa\mu} + \partial_\kappa F_{\lambda\mu})\right\} - \{\mu\leftrightarrow\nu\}\right]. \tag{3.17}$$

This result is precisely in the same form as in the bosonic string case [14]. This redefinition of the gauge field strength is again related to the redefinition of the gauge field as

$$A^{\rm red}_\mu(x) \equiv A_\mu(x) + \frac{1}{2}\zeta(1)\left(\frac{1}{1-F(x)^2}\right)^{\lambda\nu}\partial_\lambda F_{\nu\mu}(x). \tag{3.18}$$

Hence for the well-defined boundary state which is not divergent, we should put the coefficient of $\zeta(1)$ in the above redefinition to zero. This is a constraint which the gauge field in the boundary state should satisfy.

From the expression (3.18), one observes that the constraint on the gauge field is identical to the vanishing of the β function in the superstring σ model. Here we derived the constraint for the background gauge field from the consistency of the generalized boundary state. In general, string theory in some back ground field put some restriction on the background when the consistency such as the conformal invariance is required. In our case, the well-definedness of the boundary state has put such a constraint. Note that hereafter we adopt the background gauge field which satisfies the constraint $\beta[A]=0$.

The fact that the divergence (3.17) takes the same form as in the bosonic string theory has been expected from the string σ model calculation. This is because, in ref. [17] the divergence in the superstring σ model was shown to be equal to the one in the bosonic string [18] within the one-loop calculation.

Similar to the bosonic string theory [14], the divergence encoded in the field strength would be interpreted also from the superstring σ model loop calculation. The divergence in the boundary state might correspond to the one-loop divergence against the propagator on the boundary of the string worldsheet. (For details, see ref. [14].)

3.2 Correction to the D-brane action

Now we have obtained the well-defined boundary state with no divergence, let us consider the coupling between the closed string excitation and the boundary degrees of freedom. For obtaining the massless closed string coupling in the D-brane action, we extract the relevant massless part of the boundary state. Using the boundary state, we get information on the terms linear in the closed string massless modes.

Although the divergence immanent in the supersymmetric boundary state is found to be equal to the one in the bosonic string case, the finite part (the second term on the right-hand side of eq. (3.16)) has different structures. In the boundary state, the part which is relevant for the D-brane action is concerning the emission of the massless states of the closed string. The projector which extracts the relevant emission mode is studied recently in ref. [8]. In our language, the modes in V relevant for the massless emission is only the constant part (since if one excite α creation operator, the state becomes massive). In addition, the relevant part in $V^{(f)}$ is the constant mode and $b^{(-)}_{-1/2}b^{(+)}_{-1/2}$ (which is necessary only for the NS-NS sector).

It is shown in appendix A that all of these relevant modes do not appear in the second term on the right-hand side of eq. (3.16), after changing all the oscillators in V and $V^{(f)}$ into the creation operators. Especially, the constant mode which exists in the bosonic string case, stemming from V, is canceled exactly by the constant term coming from fermionic contribution $V^{(f)}$.

Therefore we conclude that, as for the part which can be extracted from the boundary state (more precisely, the couplings linear in the closed string massless modes), there is *no* $O(\alpha'\partial^2)$ *correction* to the D-brane action. This is consistent with expectation from the other works on the correction to the D-brane action, refs. [11, 12]. In these literature, it was shown that there is no $O(\alpha'\partial^2)$ correction in the other sectors (the (curvature)^2 sector, and the Born-Infeld sector with purely open string gauge fields, respectively). This property of the absence of the $O(\alpha'\partial^2)$ terms in the D-brane action seems to be universal in superstring theory.

4. Incorporation of scalar field and T-duality

In our previous paper [14] and in the former part of this paper, we have studied only the gauge field as a boundary degree of freedom. In addition to the gauge field, there exist scalar fields as another massless excitation on the D-brane. This scalar field, usually treated on an equal footing with the gauge field, represents the deformation of the D-brane.

In this section, we incorporate this scalar field into the boundary state. Similar to the gauge field, the scalar field takes an arbitrary configuration. The T-duality naturally relates this scalar field and the gauge field on the D-brane, hence we use this perturbative duality so as to check the consistency.

4.1 Incorporation of the scalar field

For simplicity, we concentrate on the bosonic string theory in sections 4.1 and 4.2. The generalization to the superstring theory is straightforward, and it will be briefly mentioned later.

Since the scalar field parameterizes the deformation of the D-brane in the static gauge, let us define the boundary state with the scalar field $\phi^i(X)$ as

$$\left|\mathcal{B}^{(b)}[\phi, A]\right\rangle \equiv \widetilde{U}[\phi]U[A]\left|B^{(b)}(F=0)\right\rangle , \tag{4.1}$$

with a translation operator

$$\widetilde{U}[\phi] \equiv \exp\left(-i\oint d\sigma P_i \phi^i(X)\right). \tag{4.2}$$

Note that the arguments of the scalar ϕ are X^μ, the tangential coordinates. When ϕ is linear in X as $\phi^i(X) = \theta^i_\mu X^\mu$, then we reproduce the result of the tilted D-brane case in ref. [15].

The boundary conditions which the above boundary state satisfies are as follows. For the transverse directions, the D-branes are now expected to be deformed to a surface specified by the scalar, and actually we have

$$\left[X^i - \phi^i(X^\mu)\right]\left|\mathcal{B}^{(b)}[\phi, A]\right\rangle = 0\,. \tag{4.3}$$

On the other hand, the boundary conditions for the tangential directions are also modified as

$$\left[\pi\left(P_\mu + P_i\partial_\mu\phi^i(X)\right) + \partial_\sigma X^\nu F_{\mu\nu}(X)\right]\left|\mathcal{B}^{(b)}[\phi, A]\right\rangle = 0\,. \tag{4.4}$$

This modification is natural in a sense that the combination $P_\mu + P_i\partial_\mu\phi^i(X)$ denotes the translation along the deformed surface of the D-brane. The ghost part is unchanged, as in ref. [14].

Both the definition (4.1) and the boundary conditions (4.3) and (4.4) can be understood through the T-duality. This duality transformation is the exchange of the sign of the right-moving oscillators : T-duality in the M-th direction is defined as $\alpha^{M(-)} \longrightarrow -\alpha^{M(-)}$. Under this transformation, X is exchanged for the conjugate momentum as

$$\partial_\sigma X^M \longleftrightarrow \partial_\tau X^M\Big(= -\pi P^M\Big)\,. \tag{4.5}$$

Now, consider the situation with vanishing scalar field on the Dp-brane. Taking the T-duality (4.5) in one of the tangential direction $\mu = p$ in the boundary condition (2.12), then we obtain new boundary conditions

$$\pi P_p + \partial_\sigma X^\nu F_{p\nu}(X) = 0\,, \tag{4.6}$$
$$\pi P_{\widetilde{\mu}} + \partial_\sigma X^{\widetilde{\nu}} F_{\widetilde{\mu}\widetilde{\nu}}(X) + \partial_\sigma X^p F_{\widetilde{\mu}p}(X) = 0\,, \tag{4.7}$$

on the T-dualized boundary state. The new tangential index $\widetilde{\mu}$ runs from 0 to $p-1$. Using a relation

$$F_{p\nu} = -\partial_\nu A_p \tag{4.8}$$

and defining new scalar field as $\phi_p \equiv -A_p$, then we reproduce eq. (4.4) for D$(p-1)$-brane and

$$\partial_\sigma (X^p - \phi^p) \, |\mathcal{B}\rangle = 0 \,. \tag{4.9}$$

This is consistent with the boundary condition in the transverse direction (4.3), and only the zero-mode part is not reproduced.[2] If one is not concerned with the zero modes, it is also possible to understand directly definition (4.1) as a result of the T-duality. Taking the T-dualities of the operator $U[A]$, then we obtain the definition of the boundary state with the scalar field (4.2).

Inspite of the introduction of the scalar field, the boundary state is still BRST invariant. In the case of the bosonic string theory, we need to verify the vanishing of the quantity $P_M \partial_\sigma X^M$ [14] for the BRST invariance. With use of the boundary conditions (4.3) and (4.4), this quantity is evaluated as

$$P_\mu \partial_\sigma X^\mu + P_i \partial_\sigma X^i = P_i(\partial_\sigma X^i - \partial_\sigma X^\mu \partial_\mu \phi^i(X)) - \frac{1}{\pi} \partial_\sigma X^\mu \partial_\sigma X^\nu F_{\mu\nu}(X) = 0 \,. \tag{4.10}$$

This BRST invariance is seen clearly from the T-duality. Actually, The operator $P_M \partial_\sigma X^M$ (and the BRST charge Q_{B}) is T-duality invariant, hence if the previous boundary state with only the gauge field is BRST invariant, then this is also the case for the one with the scalar field.

4.2 Divergence in the boundary state

Using the T-duality nature of the definition of the boundary state (4.1), as for the divergence in the boundary state, we are trivially led to the same result as in the previous paper [15]. Since the translation operator $\widetilde{U}[\phi]$ is transformed into the operator $U[A]$ by the T-duality, the procedure of changing all the operators into the creation operators is actually almost the same. All the divergences in the boundary state can be arranged in such a way that they are absorbed into the redefinition of the gauge field and the scalar field as

$$A_M^{\mathrm{red}}(x) \equiv A_M(x) + \frac{1}{4} \zeta(1) J^{LN}(\theta, F) \partial_L F_{NM}(x) \,, \tag{4.11}$$

where $J(\theta, F) \equiv 1 + \mathcal{O}(\theta, F)$. The indices $M, N, \ldots$ run through all the spacetime directions, and the gauge fields with transverse index should be understood as scalar fields. The matrix $\mathcal{O}(\theta, F)$ is defined in the same manner as ${\mathcal{O}_M}^N = {(1-F)_N}^L {\{(1+F)^{-1}\}_L}^N$, using the following definition:

$$F_{MN} \equiv \begin{pmatrix} F_{\mu\nu}(x) & \theta^i_\mu(x) \\ -\theta^j_\nu(x) & 0 \end{pmatrix}, \quad \text{where} \quad \theta^i_\mu(x) \equiv \partial_\mu \phi^i(x) \,. \tag{4.12}$$

[2]The discrepancy on the zero-mode part is owing to the fact that we are considering only the oscillator part of the T-duality transformation, (4.5). If the target space is compactified, winding modes appear and it becomes possible to treat zero modes simultaneously with oscillating modes. In this paper, uncompactified flat spacetime is assumed.

Explicitly for the scalar field, the redefinition becomes

$$\phi^i_{\rm red}(x) \equiv \phi^i(x) + \frac{1}{4}\zeta(1) J^{\mu\nu}(\theta, F)\partial_\mu\partial_\nu\phi^i(x) + \cdots . \qquad (4.13)$$

For the boundary state to be well defined, these divergences should be eliminated, therefore the background configuration of the gauge field and the scalar field should be restricted to the one which makes the coefficient of $\zeta(1)$ in eq. (4.11) equal to zero.

The only difference in calculating the divergence is that in $\widetilde{U}[\phi]$ there exists the zero mode p^i of the operator P^i, although in $U[A]$ the corresponding $\partial_\sigma X^\mu$ has no zero mode. This affects the zero mode of the transverse part of the boundary state, that has been a delta function (see eq. (2.6)).

Let us see concretely the zero-mode part. The relevant modes in the translation operator $\widetilde{U}[\phi]$ are in the following:

$$\widetilde{U}[\phi] = \exp\left(-\phi^i(x)\frac{\partial}{\partial x^i} - \frac{1}{4\pi}\left[\oint d\sigma \widetilde{X}^\mu \widetilde{X}^\nu\right]\partial_\mu\partial_\nu\phi^i(x)\frac{\partial}{\partial x^i} + \cdots\right). \qquad (4.14)$$

Here we expand the scalar field as

$$\phi^i(X) = \phi^i(x) + \widetilde{X}^\mu\partial_\mu\phi^i(x) + \frac{1}{2}\widetilde{X}^\mu\widetilde{X}^\nu\partial_\mu\partial_\nu\phi^i(x) + \cdots . \qquad (4.15)$$

Therefore the zero mode of the generalized boundary state is

$$\delta^{(9-p)}\left(x^i - \phi^i(x)\right) - \frac{1}{4\pi}\left[\oint d\sigma \widetilde{X}^\mu\widetilde{X}^\nu\right]\partial_\mu\partial_\nu\phi^i(x)\frac{\partial}{\partial x^i}\delta^{(9-p)}\left(x^i - \phi^i(x)\right) + \cdots , \quad (4.16)$$

where the last omitted part consists of higher order terms.[3] The expression of the first term in the above representation (4.16) is due to the first term in the exponent in eq. (4.14). Evaluating the divergence in the second term in eq. (4.16) with use of the boundary condition analogous to (3.13) now with the constant field strength (4.12), then we see that the result is

$$-\frac{1}{4\pi}\cdot\pi\zeta(1)J^{\mu\nu}\cdot\partial_\mu\partial_\nu\phi^i(x)\frac{\partial}{\partial x^i}\delta^{(9-p)}\left(x^i - \phi^i(x)\right). \qquad (4.17)$$

This divergence can be absorbed into the delta function part in the boundary state, with the redefinition (4.13) as

$$\delta^{(9-p)}\left(x^i - \phi^i_{\rm red}(x)\right). \qquad (4.18)$$

For general configurations of A_μ and ϕ^i, the correction to the D-brane action can be calculated using dimensional reduction (with the identification $A_p = -\phi^p$),

[3]Although the second term in eq. (4.16) proportional to $\partial\partial\phi$ corresponds to the ∂F mode in $U[A]$ by T-duality, this term contains already the derivative acting on the delta function, thus the number of the derivatives on the field strength $(\partial\phi)$ are already two. Therefore, the term in proportion to $\partial\partial(\partial\phi)$ or $\partial(\partial\phi)\partial(\partial\phi)$ are higher order terms.

due to the T-duality. However, as mentioned above, only the zero mode part of the translation operator $\widetilde{U}[\phi]$ is different from T-dualized $U[A]$, thus from this zero mode there appears a new finite part which contributes to the D-brane action. The relevant term from the second term in eq. (4.16) is

$$\frac{1}{4} J^{\rho\lambda} J^{\mu\nu} \partial_\nu \, \partial_\rho \, \phi^i(x) \frac{\partial}{\partial x^i} \, \delta^{(9-p)} \left(x^i - \phi^i(x) \right) \cdot \alpha^{(+)}_{-1\lambda} \alpha^{(-)}_{-1\mu} \, . \tag{4.19}$$

When contracting with the closed string massless modes, this contribution gives a new term in the correction to the bosonic D-brane action, in addition to the contributions from the dimensional reduction of the gauge field from ten-dimension.

4.3 Supersymmetric case and correction to the D-brane action

As for the supersymmetric boundary state studied in section 2, the scalar fields can be associated in the same manner. Adopt the same redefinition $\alpha^{(-)} \longrightarrow -\alpha^{(-)}$ and $\phi^i(X) \equiv -A^i(X)$, and then for the fermionic coordinates, define the T-duality transformation as

$$\psi_- \longrightarrow -\psi_- \, . \tag{4.20}$$

All of the calculation of the divergence in the boundary state can be read in correspondence with section 3 through the T-duality, except for the zero modes of the operator $\widetilde{U}[\phi]$ considered in the previous subsection. (Note that fermionic zero mode in the R-R sector does not make mischief.) As for the divergence, this zero mode contribution precisely gives the redefinition of the scalar field in the delta function, which results in no correction. Although in the bosonic string theory this zero mode contribution yields the correction to the D-brane action (4.19), in superstring case this correction does not appear since the corresponding excitation is massive.

As seen in section 3.2, there is no $O(\alpha'\partial^2)$ correction to the D-brane action in the superstring theory in the case of non-trivial gauge field configuration. Hence in the superstring case, even if one incorporates the general brane deformation specified by $\phi(x^\mu)$, the D-brane action is not corrected to this order.

5. Conclusion and discussion

In this paper we have constructed generalized supersymmetric boundary state which incorporates arbitrary configurations of massless fields on the D-brane. In order to introduce such arbitrariness, we follow ref. [14], where the bosonic boundary state was generalized by a gauge transformation of a string field theory which is an analogue of a string σ model gauge transformation of closed string theory.

The newly defined supersymmetric boundary state is BRST invariant, and obeys the non-linear boundary conditions (2.23) and (2.24) which are in the same form as derived in a superstring σ model [17]. Though this BRST invariance is verified for arbitrary configuration of the background gauge field, the boundary state contains short distance divergences originating in the products of the coordinate fields on the worldsheet. For obtaining well-defined boundary state, the divergent entry should be eliminated, so we restrict the background gauge field configuration. This constraint on the gauge field has been found to be identical to the conformal invariance ($\beta(A) = 0$) in a superstring σ model loop calculation. This has been checked at least within the next-to-leading order ($O(\alpha'\partial^2)$ correction, corresponding to the one-loop calculation in the σ model). After this subtraction, we have extracted finite corrections to the D-brane action at this order, and have found that there exists no correction of $O(\alpha'\partial^2)$ to the coupling linear in the closed string massless modes. For the R-R sector this property of no-correction is expected from the fact that the coupling is relevant for the anomaly cancellation. On the other hand, for the NS-NS sector the result is non trivial. The absence of the $O(\alpha'\partial^2)$ correction to the D-brane action seems to be universal (see refs. [11, 12, 19]).

The T-duality transformation relates Dirichlet and Neumann directions, hence it interchanges the gauge field with the scalar field on the D-brane. This T-duality has enabled us to incorporate also the general configuration of the scalar field into the boundary state. Since the whole massless degrees of freedom are now incorporated in the boundary state formalism, it would be possible to analyze the non-trivial configuration of the fields on the brane and various D-brane configuration directly by the conformal field theory.

Taking a BPS configuration which was previously studied as BIon [20, 21, 22], the divergence in the boundary state vanishes and one obtains a boundary state for branes ending on another brane. Especially adopting a BIon solution which represents a D-string stuck to a D3-brane, in appendix B we have evaluated the boundary state near the spike, and showed that in this region the form of the boundary state approaches actually to the one of the D-string. This BIon boundary state are to be examined from various aspects in details.

Another example for the application is the string junctions [23] which have been already realized in two-dimensional gauge theory on D-strings [24]. Therefore it is possible to discuss the junctions with a single boundary state. Or, using other BIon configurations, one can study fundamental strings stuck to a D-brane. In this case, boundary states representing fundamental strings[4] appear, and it is interesting to study how this boundary states express the mechanism of the joining-splitting process of the fundamental strings.

On the other hand, most of intriguing brane configurations are related to non-abelian configurations of Yang-Mills-Higgs theory. One of the examples belonging to this category is string junctions terminated on D3-branes (1/4 BPS dyons in super Yang-Mills theory [25]). A non-abelian version of the generalized boundary state

[4]See ref. [8] for a related discussion.

would be obtained with use of a path-ordered unitary operator $U[A]$. Evaluation of this kind of operator seems to be difficult technically. These remain to be studied in the future works.

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A. Calculation of the divergence in the boundary state

In this appendix, we present the evaluation of the generalized boundary state. As mentioned in section 3, for extracting the couplings between closed string excitations and boundary degrees of freedom, the boundary state should be in a form where all the oscillators are written only by creation operators. For simplicity, we do not consider the scalar field in this appendix.

We calculate the divergences in the boundary state by changing all the oscillators in V and $V^{(\mathrm{f})}$ (eqs. (3.8) and (3.9)) into creation operators explicitly, by using the boundary conditions (3.13) and (3.14) which hold on the boundary state $|B(F(x))\rangle$. We keep only terms up to $O(\alpha'\partial^2)$ (at most two derivatives on the field strength F), therefore the quantity to be evaluated is

$$1 + (V-1) + (V^{(\mathrm{f})}-1) + V_{\mathrm{mix}}\,, \tag{A.1}$$

where the last term V_{mix} contains contributions from both the bosonic part and the fermionic part,

$$\begin{aligned} V_{\mathrm{mix}} \equiv & \left[\frac{-i}{3\pi}\partial_\rho F_{\nu\mu}(x) \oint d\sigma \partial_\sigma \widetilde{X}^\mu \widetilde{X}^\nu \widetilde{X}^\rho\right] \times \\ & \times \left[\frac{1}{8\pi}\partial_a F_{bc}(x) \oint d\sigma (\psi_+ \pm i\psi_-)^b (\psi_+ \pm i\psi_-)^c \widetilde{X}^a\right]. \end{aligned} \tag{A.2}$$

We shall evaluate these three terms $(V-1)$, $(V^{(\mathrm{f})}-1)$ and V_{mix}, respectively in the following:

1. For the bosonic part $V-1$, we already have the result in ref. [15], since the boundary condition for the bosonic oscillators in the supersymmetric boundary state is the same as in the bosonic boundary state. The result is as follows: first, the $\zeta(1)$ divergences in V can be completely absorbed into the redefinition of

the gauge field (or the gauge field strength) of the bosonic part of the boundary state $|B(F(x))\rangle$. This redefinition is given by eq. (3.17). Second, the rest finite corrections to the boundary state is

$$\frac{1}{64}\partial_\gamma F_{\alpha\beta}\partial_\rho F_{\mu\nu}\left[J^{\rho\gamma}J^{\beta\nu}(J^{\mu\alpha}+J^{\alpha\mu})\right]\Big|B\Big(F(x)\Big)\Big\rangle + \\ + \begin{bmatrix}\text{even number of}\\ \alpha\text{ oscillators}\end{bmatrix}\Big|B\Big(F(x)\Big)\Big\rangle\,. \tag{A.3}$$

The second term in the above equation does not contribute to the D-brane effective action,[5] since these excitations are related to the coupling between the boundary and the massive excitations of the closed string: $(\alpha^\dagger)^2+(\alpha^\dagger)^4+(\alpha^\dagger)^6$. Hence the relevant correction is the first term in eq. (A.3).

2. The last term V_{mix} in (A.1) does not contribute both to the divergence in the boundary state and to the D-brane effective action. A possible contraction which may bring out the divergence is

$$\oint d\sigma\partial_\sigma\widetilde{X}^\mu\widetilde{X}^\nu\widetilde{X}^\rho\oint d\sigma(\psi_+\pm i\psi_-)^b(\psi_+\pm i\psi_-)^c\widetilde{X}^a\,. \tag{A.5}$$

(This is because if one contract two of the three X's in the first integral, the rest single X has no zero mode and therefore the whole quantity vanishes. This is also the case for the fermion contraction.) Evaluating the contraction (A.5), one can easily see that there is no divergence $\zeta(1)$. Additionally, there is no contribution to the D-brane, since the massless excitation part of the boundary state does not stem from this V_{mix}, even after the contraction in the way (A.5).

3. Finally, for the fermionic part $(V^{(\text{f})}-1)$, we evaluate three non-trivial terms in eq. (3.9).

   (a) First, the term proportional to ∂F (the first line in eq. (3.9)) has no divergence and it is safely changed to the form written only by the creation operators. This is because the contraction between two ψ's makes no sence due to the fact that $\widetilde{X}$ has no zero mode.

   (b) Second, let us see that the term proportional to $\partial\partial F$ (the second line in eq. (3.9)) contains divergence. Changing all the α oscillators in two $\widetilde{X}$'s

[5] In the boundary state, the terms which contribute to the D-brane action is of the form

$$\text{const.}\times\Big|B\Big(F(x)\Big)\Big\rangle \qquad \text{or} \qquad b^{(+)\dagger}b^{(-)\dagger}\Big|B\Big(F(x)\Big)\Big\rangle\,. \tag{A.4}$$

Note that this is not the case for the bosonic string. See ref. [15].

into creation operators, then we have

$$\oint d\sigma(\psi_+ \pm i\psi_-)^\mu(\psi_+ \pm i\psi_-)^\nu \widetilde{X}^\rho \widetilde{X}^\delta = \left[(\alpha^\dagger)^2 + \frac{1}{4}\zeta(1)\left(J^{\rho\delta} + J^{\delta\rho}\right)\right] \times$$
$$\times \oint d\sigma(\psi_+ \pm i\psi_-)^\mu(\psi_+ \pm i\psi_-)^\nu \,. \tag{A.6}$$

This equality holds only on the boundary state $|B(F(x))\rangle$. The rest fermionic contraction is

$$\oint d\sigma \overbrace{(\psi)^\mu (\psi)}^{}{}^\nu \widetilde{X}^\rho \widetilde{X}^\delta, \tag{A.7}$$

however this does not give any divergence. Therefore, one sees the single divergent term in eq. (A.6) from the $\partial\partial F$ part in $(V^{(\mathrm{f})} - 1)$. Referring to the definition of the exponent R of the boundary state, eq. (2.18), this $\zeta(1)$ divergence in (A.6) can be absorbed into the redefinition of the gauge field strength in the boundary state as

$$F^{\mathrm{red}}_{\mu\nu}(x) \equiv F_{\mu\nu}(x) + \frac{1}{2}\zeta(1)\left(\frac{1}{1+F}\right)^{\rho\delta} \partial_\rho\, \partial_\delta F_{\mu\nu}\,. \tag{A.8}$$

This is a part of the total redefinition of the gauge field (3.17). The finite part after the redefinition of the gauge field strength consists of the $(\alpha^\dagger)^2$ term in eqs. (A.6) and (A.7). Both of these terms contain two α's, hence do not couple to the massless modes of the closed superstring. Therefore, the $\partial\partial F$ part does not give any correction to the D-brane action.

(c) The evaluation of the final term in $(V^{(\mathrm{f})} - 1)$,

$$\oint d\sigma \left(\widetilde{\psi}_+ \pm i\widetilde{\psi}_-\right)^\mu \left(\widetilde{\psi}_+ \pm i\widetilde{\psi}_-\right)^\nu \widetilde{X}^\rho \times$$
$$\times \oint d\sigma \left(\widetilde{\psi}_+ \pm i\widetilde{\psi}_-\right)^b \left(\widetilde{\psi}_+ \pm i\widetilde{\psi}_-\right)^c \widetilde{X}^a\,, \tag{A.9}$$

is found to be rather complicated, as in the case of the bosonic string [15]. For simplicity, we consider only $\widetilde{\psi}$ which is non zero modes of the fermionic operator ψ. (The zero modes of ψ exists only in the R-R sector, and they will be treated separately later.) We change all the oscillators in this (A.9) into creation operators, using the boundary conditions (3.13) and (3.14). After some straight forward calculation, the result is found as

$$(\mathrm{A.9}) = \mathrm{c} + \left(\mathrm{b}^\dagger\right)^2 + (\text{other terms}) \tag{A.10}$$

on the boundary state $|B(F(x))\rangle$. Here the first constant c term and the second term quadratic in b contain $\zeta(1)$ divergence as seen in the following,

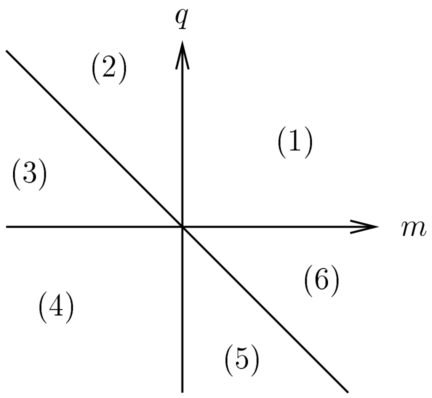

Figure 1: The region for summation in eq. (A.11).

and the rest "other terms" consisting of $(b^\dagger)^2(\alpha^\dagger)^2$, $(b^\dagger)^4$ and $(b^\dagger)^4(\alpha^\dagger)^2$ have finite coefficients. Furthermore, these "other terms" couple to the massive excitations of the closed superstring and thus have no contribution to the D-brane action. The first constant term c is given as follows:

$$\begin{aligned}(2\pi)^2\Bigg[&\sum_{(1)}\frac{1}{4m}J^{\rho a}\left(J^{\nu c}J^{b\mu}-J^{\mu c}J^{b\nu}\right)+\\ &+\sum_{(2)}\frac{-1}{4m}J^{a\rho}\left(J^{\nu c}J^{b\mu}-J^{\mu c}J^{b\nu}\right)+\\ &+\sum_{(3)}\frac{-1}{4m}J^{a\rho}\left(-J^{\nu c}J^{\mu b}+J^{\mu c}J^{\nu b}\right)+\\ &+\sum_{(4)}\frac{-1}{4m}J^{a\rho}\left(J^{c\nu}J^{\mu b}-J^{c\mu}J^{\nu b}\right)+\\ &+\sum_{(5)}\frac{1}{4m}J^{\rho a}\left(J^{c\nu}J^{\mu b}-J^{c\mu}J^{\nu b}\right)+\\ &+\sum_{(6)}\frac{1}{4m}J^{\rho a}\left(-J^{c\nu}J^{b\mu}-J^{c\mu}J^{b\nu}\right)\Bigg].\end{aligned}\qquad\text{(A.11)}$$

The region for summation is depicted in figure 1, where the index q stems from the fermionic contraction. The summation of $1/m$ in each region is given as

$$\sum_{(1)}\frac{1}{m}=\begin{cases}0 & \text{(NS-NS)},\\ \zeta(1)\zeta(0) & \text{(R-R)},\end{cases}$$

$$\sum_{(2)}\frac{1}{m}=\begin{cases}\zeta(0) & \text{(NS-NS)},\\ -\zeta(1)\zeta(0)+\zeta(0) & \text{(R-R)},\end{cases}$$

$$\sum_{(3)} \frac{1}{m} = \begin{cases} -\zeta(0) & \text{(NS-NS)}\,, \\ \zeta(1) - \zeta(0) & \text{(R-R)}\,, \end{cases}$$

$$\sum_{(i+3)} \frac{1}{m} = -\sum_{(i)} \frac{1}{m} \quad \text{for} \quad i = 1, 2, 3\,. \tag{A.12}$$

The difference between the NS-NS sector and the R-R sector stems from the fact that the lattice point to be summed in the region consists of integer lattice (the NS-NS sector) or half-odd integer lattice (the R-R sector) along the direction of axis q. For the NS-NS sector, there is no lattice point on the line $q + m = 0$ in figure 1, while for the R-R sector, lattice points on this line will give the zero mode contribution which will be dealt with later. One can see from eq. (A.12) that the $\zeta(1)$ divergence exists only in the R-R sector, and finite terms (after regularizing as $\zeta(0) = -1/2$) for the R-R sector and the NS-NS sector are the same. Substituting the summation (A.12) into (A.11), the $\zeta(1)$ divergent term in c in eq. (A.10) is found as

$$(2\pi)^2 \zeta(1) J^{\rho a} J^{b\mu} (J^{c\nu} - J^{\nu c}) \tag{A.13}$$

which exists only in the R-R sector, and the finite term common in both the R-R and NS-NS sector is

$$(2\pi)^2 \frac{1}{2} J^{\rho a} J^{c\nu} \left(J^{b\mu} + J^{\mu b} \right)\,. \tag{A.14}$$

Referring to eq. (2.28), only in the R-R sector there is a front factor in the boundary state for the fermionic sector (in the case of constant field strength). Thus the divergence (A.13) in front of the boundary state $|B(F(x))\rangle$ can be absorbed into the front normalization factor of the boundary state through redefinition of the gauge field strength as

$$\begin{aligned} F^{\rm red}_{\mu\nu}(x) \equiv F_{\mu\nu}(x) + \frac{1}{16}\zeta(1) \times \\ \times \Big[\left\{ J^{\lambda\rho} \partial_\nu F_{\rho\delta} J^{\delta\kappa} (\partial_\lambda F_{\kappa\mu} + \partial_\kappa F_{\lambda\mu}) \right\} - \{\mu \leftrightarrow \nu\} \Big]. \end{aligned} \tag{A.15}$$

This is the last term in the redefinition (3.17). On the other hand, the finite contribution (A.14) results in a final form

$$-\frac{1}{64} \partial_\gamma F_{\alpha\beta}\, \partial_\rho F_{\mu\nu} \left[J^{\rho\gamma} J^{\beta\nu} (J^{\mu\alpha} + J^{\alpha\mu}) \right] \Big| B\Big(F(x)\Big) \Big\rangle\,. \tag{A.16}$$

This is common for both the NS-NS and R-R sector. Furthermore, one observes that this finite constant factor exactly cancels the finite constant factor stemming from the bosonic part (A.3).

JHEP04(2000)023

Second, we check that the divergence in the $(b^\dagger)^2$ term in eq. (A.10) can be absorbed also by the redefinition (A.15). The explicit expression of the $(b^\dagger)^2$ term in eq. (A.10) is

$$
\begin{aligned}
(2\pi)^2\Bigg[\sum_{(1)}\frac{1}{m}J^{a\rho}\Big(&-J^{d\nu}J^{\mu c}J^{bf}b^{(-)}_{-qd}\,b^{(+)}_{-qf}-\\
&-J^{c\mu}J^{\nu f}J^{db}b^{(-)}_{-m-qd}\,b^{(+)}_{-m-qf}\Big)+\\
+\sum_{(1)}\frac{-1}{m}J^{a\rho}\Big(&-J^{dc}J^{\mu f}J^{b\nu}b^{(-)}_{-qd}\,b^{(+)}_{-qf}-\\
&-J^{cf}J^{\nu b}J^{d\mu}b^{(-)}_{-m-qd}\,b^{(+)}_{-m-qf}\Big)+\\
+\sum_{(3)}\frac{-1}{m}J^{a\rho}&\left(J^{dc}J^{\nu b}J^{\mu f}b^{(-)}_{-qd}\,b^{(+)}_{-qf}\right)+\\
+\sum_{(6)}\frac{1}{m}J^{a\rho}&\left(J^{dc}J^{\nu b}J^{\mu f}b^{(-)}_{-m-qd}\,b^{(+)}_{-m-qf}\right)\Bigg].
\end{aligned}
\tag{A.17}
$$

In obtaining this expression, we have used the antisymmetric property of the indices $(b\leftrightarrow c)$ and $(\mu\leftrightarrow\nu)$, and change of the region for summation: (i)$\leftrightarrow$(i+3) with $(m,q)\leftrightarrow(-m,-q)$ simultaneously. Let us extract the $\zeta(1)$ divergence in this expression (A.17). For example, in the last term in (A.17), there is a $\zeta(1)$ divergence in the summation $1/m$ in the region (6), as seen if we fix the excitation number $-m-q$ of the oscillator b. In this manner, we obtain the divergence in $(b^\dagger)^2$ in eq. (A.10) as

$$
2(2\pi)^2\zeta(1)\left[J^{d\mu}J^{\nu c}J^{bf}-J^{db}J^{\nu c}J^{\mu f}\right]\sum_{q>0}b^{(-)}_{-qd}\,b^{(+)}_{-qf}\,. \tag{A.18}
$$

It is easy to show that this divergence can be absorbed into the exponent of the boundary state (2.26) by the redefinition (A.15).

The finite contribution to the D-brane action from (A.17) should be of the form $b^{(-)}_{-1/2}\,b^{(+)}_{-1/2}$ in the NS-NS sector. Extracting that part, the coefficient of the term is turned out to be zero. Thus there is no contribution to the D-brane action from this $(b^\dagger)^2$ term.

Finally, we consider the zero mode contribution in the term (A.9) in the R-R sector. Written explicitly the coefficients, the zero mode contribution is

$$
\begin{aligned}
4\times\frac{1}{2}\left(\frac{1}{8\pi}\right)^2\partial_\rho F_{\mu\nu}\partial_a F_{bc}\oint d\sigma\,\theta^\mu\left(\widetilde{\psi}_+\pm i\widetilde{\psi}_-\right)^\nu\widetilde{X}^\rho\times\\
\times\oint d\sigma\,\theta^b\left(\widetilde{\psi}_+\pm i\widetilde{\psi}_-\right)^c\widetilde{X}^a\,.
\end{aligned}
\tag{A.19}
$$

The front factor 4 is due to the antisymmetric nature of the indices of F. Changing all the oscillators in (A.19) with use of the boundary conditions (3.13) and (3.14), we obtain

$$\frac{-1}{32}\zeta(1)\partial_\rho F_{\mu\nu}\partial_a F_{bc}\theta^\mu\theta^b\left(J^{\nu c}J^{a\rho}-J^{c\nu}J^{\rho a}\right)+\left(\alpha^\dagger\right)^2+\left(b^\dagger\right)^2+\left(\alpha^\dagger\right)^2\left(b^\dagger\right)^2 . \tag{A.20}$$

The last three terms do not contain any divergence, and have no contribution to the D-brane action. The first divergent term can be absorbed by the redefinition of the field strength (A.15), into the fermionic zero mode part of the boundary state (2.27).

Summing up all contributions, we conclude that all the divergences in the boundary state can be absorbed into the redefinition of the gauge field strength (3.17), and the rest finite contributions relevant for the D-brane action vanishes.

B. Boundary state for brane ending on brane

Well-defined configurations of the introduced gauge field and scalar field are the ones in which the divergent part of the redefinition (3.17) (and (4.13)) vanishes. Therefore for studying the well-defined boundary state, one must substitute to A_μ and ϕ^i the solutions of this constraint which is identical with the leading-order equations of motion derived from the open superstring σ model approach.

One of the interesting solution is the BIon configuration (or called "spike soliton") [20, 21]. This configuration is known to be a solution of the equations of motion corrected to all order in the derivative expansion [26]. Therefore, if one assumes that the divergence in the boundary state exactly coincides with the divergence in the string σ model to all order, then this BIon configuration is the most natural among nontrivial configurations of A_μ and ϕ^i, for a well-defined boundary state. Another interesting respect concerning this solution is that this BIon configuration represents F-strings (or D-strings) ending on a D-brane [20, 22]. Hence adopting the BIon solution we obtain boundary states representing branes ending on another D-brane.

Let us consider a specific example of a D-string stuck to a D3-brane. The corresponding BIon solution is the BPS configuration for a point magnetic charge in the worldvolume theory on the D3-brane:

$$B_a=\partial_a\phi^9(X) \quad \text{with} \quad \phi^9=\frac{1}{r}, \qquad r\equiv\sqrt{(X^1)^2+(X^2)^2+(X^3)^2}\,, \tag{B.1}$$

where B_a is the magnetic field with $a=1,2,3$. Let us evaluate this singular operator $1/r$ by α' expansion. Put the center of mass of the string attached to this boundary state at $x^1=\epsilon$ and $x^2=x^3=0$. Then expand the magnetic field around the center of mass as

$$B_1=-\frac{1}{\epsilon^2}+\text{oscil.}\,, \quad B_2=0+\text{oscil.}\,, \quad B_3=0+\text{oscil.} \tag{B.2}$$

We evaluate the effect of the constant mode of the magnetic field. The contribution of the center of mass becomes

$$F_{MN} = \begin{pmatrix} 0 & 0 & 0 & 1/\epsilon^2 \\ 0 & 0 & -1/\epsilon^2 & 0 \\ 0 & 1/\epsilon^2 & 0 & 0 \\ -1/\epsilon^2 & 0 & 0 & 0 \end{pmatrix}, \tag{B.3}$$

where the each column and row denote the direction along x^1, x^2, x^3, x^9, respectively. Calculating the matrix $\mathcal{O}$ from this F and taking the limit $\epsilon \to 0$ (in this limit we are approaching to the D-string region, along the conjecture in ref. [20]), then we have

$$\mathcal{O} \to \mathrm{diag}(-1, -1, -1, -1)\,. \tag{B.4}$$

This means that the original Neumann directions x^1, x^2, x^3 change their signs in front of the bilinear combination of the oscillators in the exponent of the boundary state (see eq. (2.5)) and become Dirichlet directions, while the 9-th (originally Dirichlet) direction also changes the sign so as to become a Neumann type. This result does not depend on where we expand the magnetic field, so long as we take the limit of approaching to the singular point $X_a \sim 0$. Therefore we have examined that the singular center of the BIon corresponds to a D-string which extends to the 9-th direction.

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JHEP04(2000)023

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JHEP04(2000)023